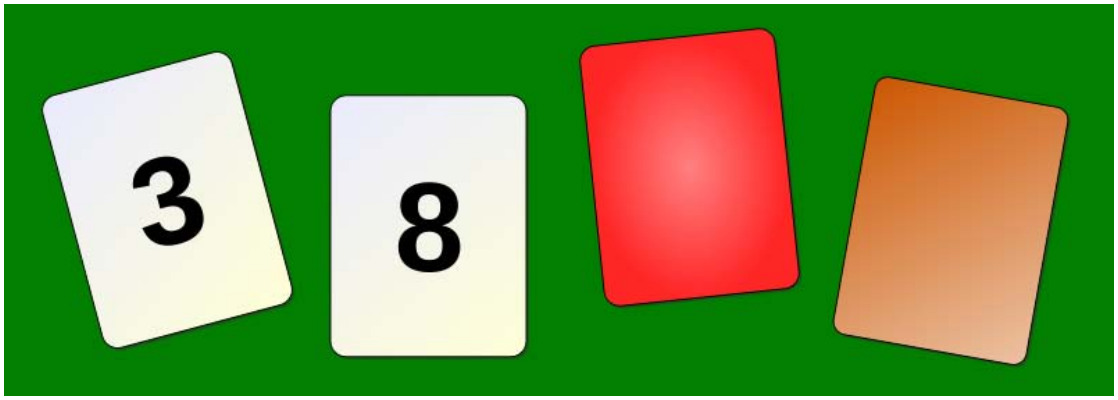
MA10209 – Week 1 Tutorial

B3/B4, Andrew Kennedy

Just for fun…



- Claim:
  If a card shows an even number on one face, then the opposite face is red.
- Which cards must be turned over to test the truth of the statement?
  - Picture: Life of Riley

Tutorial	Friday 14:15 in 8W 2.29 (weeks 1-11)
Exercise Sheets	submit solutions to folders in 4W level 1 by 13:00 each Monday I will return marked work on the Friday of the same week. (part solutions welcome)
Questions etc.:	email me, preferably by Thursday night aik22@bath.ac.uk
Useful resources:	course notes exercise sheets course text people.bath.ac.uk/aik22/ma10209

Exercise Sheet 1

- Sets
- Intersections & Unions
- Maps

  - Try to have a look at the sheet before coming to tutorials, even if it's only reading each question to make sure you understand what it's asking

  - If you get stuck, email me! aik22@bath.ac.uk
    Though remember that these questions are designed to make you think.

Notable Sets

$\mathbb{R}$	integers	$\{\}$
$\mathbb{N}$	rational numbers	$\{a+bi \mid a,b \in \mathbb{R}\}$
$\emptyset$	empty set	$\{1,2,3,\ldots\}$
$\mathbb{C}$	complex numbers	$\{\frac{a}{b} \mid a,b \in \mathbb{Z}\}$
$\mathbb{Z}$	natural numbers	$\{\ldots,-2,-1,0,1,2,\ldots\}$
$\mathbb{Q}$	real numbers	

- Match the symbol to the name of the set, and (where applicable) its definition in set notation.

Notable Sets (Answers)

symbol	set name	set notation
$\emptyset$	empty set	$\{\}$
$\mathbb{N}$	natural numbers	$\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	integers	$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
$\mathbb{Q}$	rational numbers	$\{\frac{a}{b} \mid a, b \in \mathbb{Z}\}$
$\mathbb{R}$	real numbers	
$\mathbb{C}$	complex numbers	$\{a + bi \mid a, b \in \mathbb{R}\}$

Intersections and unions

- Find non-empty sets A and B such that the following hold:
  1. $A \cup B = \{1, 2, 3, 4, 5\}; \qquad |A \cap B| = 2.$
  2. $A \cap B = A; \qquad A \neq B.$
  3. $A \cap B = \{2, 3\}; \qquad A \cup B = \{1, 2, 3, 4\};$
     $|A| = |B| = 3.$

Maps and composition

$$\text{Let } f, g : \mathbb{R} \to \mathbb{R} \text{ be maps with } f(x) = 2x^2 \text{ and } g(x) = \frac{x}{2}.$$

- Find the following:

$$f(2) \qquad g(4)$$

$$f \circ g(2) \qquad g \circ f(2)$$

- Show that $g \circ f \neq f \circ g$.

Maps and composition (Answers)

Let $f, g : \mathbb{R} \to \mathbb{R}$ be maps with
$f(x) = 2x^2$ and $g(x) = \frac{x}{2}$.

- Find the following:

$$f(2) = 8 \qquad g(4) = 2$$

$$f \circ g(2) = 2 \qquad g \circ f(2) = 4$$

Consider $y \in \mathbb{R}$. Since $f \circ g(y) = \frac{y^2}{2}$
and $g \circ f(y) = y^2$ are different,

$$f \circ g \neq g \circ f.$$

Domain & Co-domain

- What's wrong with this map?

$$\text{Let } f : \mathbb{Z} \to \mathbb{N} \text{ be a map, with } f(x) = x.$$

- How could we fix this?

Domain & Co-domain

- What's wrong with this map?

$$\text{Let } f : \mathbb{Z} \to \mathbb{N} \text{ be a map, with } f(x) = x.$$

- How could we fix this?

$$\text{Let } f : \mathbb{N} \to \mathbb{N} \text{ be a map, with } f(x) = x.$$

$$\text{Let } g : \mathbb{Z} \to \mathbb{Z} \text{ be a map, with } g(x) = x.$$

$$\text{Let } h : \mathbb{Z} \to \mathbb{N} \text{ be a map, with } h(x) = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \\ 1 & \text{if } x = 0 \end{cases}.$$

Exercise Sheet 1 - overview

- Q1 – if the set has a symbol, e.g. $\mathbb{N}$, then use that.
  Otherwise, list the elements in set notation.
- Q2 – draw pictures if it helps, then explain your answer.
- Q3-4 – similar to previous slides.

$$\text{The identity map } \mathrm{Id}_{\mathbb{R}} \text{ is defined as}$$
$$\mathrm{Id}_{\mathbb{R}} : \mathbb{R} \to \mathbb{R}, \qquad x \mapsto x.$$

Exercise Sheet 1 - overview

- Q5 – requires a bit more thought.

  (d) wouldn't be overly interesting if the answer was no!

- Q6 – 'give a geometric meaning':

  (a) - (c) will be shapes in the plane;
  (d) & (e) will be transformations;
  (f) & (g) will be shapes in 3D space.

  A picture is great, but try to describe the shape in words

Exercise Sheet 1 - overview

- Q7 – a lot of thinking required.
- Q8 – (a) isn't bad: see Q6.
  (b) try to get the idea down on paper, even if you can't express it mathematically.
  (c) consider how you might go about listing the various possible graphs.
- Q9 – the binomial expansion is

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k .$$

How to write mathematics

This doesn't apply if you're doing rough work, but if you're submitting work, or in an exam, then try to be clear.

- *How to Think Like a Mathematician*, Kevin Houston
  - http://www.kevinhouston.net/httlam.html includes two free downloadable chapters on How to Write Mathematics
- Some pointers
  - Write in sentences – include a capital letter and a full stop.
  - Explain what you're doing.
  - Equals means equals.
  - Use words/symbols appropriately.